# Workshop: “Calibration of yield curves in Mexico, with collaterals in different currencies"

September 2018

# Introduction

- The theories and models that are used for the valuation of financial instruments in general, consider the following assumptions:
  - The market is free of arbitrage.
  - There are no short sales restrictions.
  - There are no frictions in the markets, such as transaction costs and taxes.
  - There is a risk-free rate at which intermediaries can borrow or lend money, without any limitation.
  - The counterparties participating in these transactions are not exposed to a default risk.

***However, these assumptions do not reflect the reality of the financial markets!***

# I. Pre-crisis

- Prior to the credit crisis of 2007, counterparty risk was managed through a set of traditional credit limits (lines of credit and operation). Funding costs were not explicitly considered in determining the prices of derivative instruments.
- Interbank rates (Ibor) were considered as "risk free rates" and were used to discount derivative instruments. Convenience that was consistent with the assumption that banks do not go bankrupt.
- In 1988 the Basel I agreement was published and in 2004 Basel II; however, capital management was considered a back office function.
- Therefore, at that time, many financial institutions did not have sufficient reserves to face a liquidity or credit crisis.

# II. Post-crisis

- An important event that marked a change in the valuation of derivatives was the collapse of Lehman Brothers in 2008 and of many other financial institutions with high credit quality. From that moment, a process of "correction" was initiated in the methodologies and valuation assumptions.
- It became clear that the non-arbitrage assumptions were not fulfilled and could be broken due to the existence of counterparty and liquidity risks.
- From that moment:
  - The practice to discount the cash flows accruing to derivative instruments was modified, leaving Ibor rates as risk free rates.
  - Credit and liquidity costs began to be considered in the valuation of derivative instruments: CVA, DVA, FVA, KVA, ....
  - A greater number of financial contracts were begun under collateral agreements (CSA), in order to mitigate the credit risk.

# II. Post-crisis

- Credit Support Annex (CSA)
  - It is a legal document that provides credit protection to counterparties, since it establishes the rules that govern the role of collateral in derivative transactions.
  - Specifies the assets and currency that may be considered as collateral, the minimum transfer amounts, the levels of credit limits ("threshold"), among others.
  - In a standard CSA, the collateral is in the same currency as the one in which the derivative transaction is made. On the contrary, in a non-standard CSA the collateral can be constituted in a different currency.
  - This difference in CSA contracts has a significant impact on the valuation of the derivatives that are traded, since the cash flows under a non-standard CSA must be projected and discounted based on rates adjusted to the collateral currency.

# III. Valuation

- The approach for the valuation, in general of the derivative financial instruments, and in particular on the derivatives (swaps) on interest rates, consists of:
  - The cash flows to be delivered and / or received are referenced to interest rates that are denominated in the same currency.
  - Floating cash flows are projected based on a forward curve that is inferred from the spot zero coupon curve.
  - Each cash flow, to be delivered or received in the future, is discounted using a zero-discount coupon curve.
  - The zero-coupon curves are estimated from the information of the yield curves quoted in the market.

# III. Valuation

- The differences in the valuation of the derivatives, in the pre and post crisis environments are:

| Pre-crisis | Post-crisis |
|---|---|
| It is not considered a collateral | Contracts agreed upon under CSA agreements are considered.<br>Collaterals are deposited in different currencies |
| <u>Single curve approach:</u> The forward curve with which future cash flows are calculated is estimated from the zero-coupon curve used to discount flows<br>Projection curve = Discount curve | <u>Multi-curves approach:</u> The zero-coupon forward and discount curves are obtained implicitly from calibrating the rates corresponding to the currency of the collaterals<br>Projection curve ≠ Discount curve |

# III. Valuation

- Bootstrapping
  - The construction of the zero-coupon curves, which are used to discount the cash flows, from the yield curves that are traded in the market, is done using the technique called bootstrapping.
  - This method is based on the fact that:
    - The theoretical price of a bond is equal to the sum of the discounted cash flows with the zero-coupon rate at the term of each flow or with the yield rate at the term of the bond.
    - That a zero-coupon curve can be constructed from a set of bond prices with identical contractual characteristics, but with different maturities.
  - The bootstrapping process starts by estimating zero coupon rates at the shortest terms; and it continues sequentially towards longer maturities. In the process, the missing information can be estimated using interpolation.
- **Exercise.** Obtain the zero-coupon curve of a set of M bond prices at different maturities.

# IV. Single curve approach

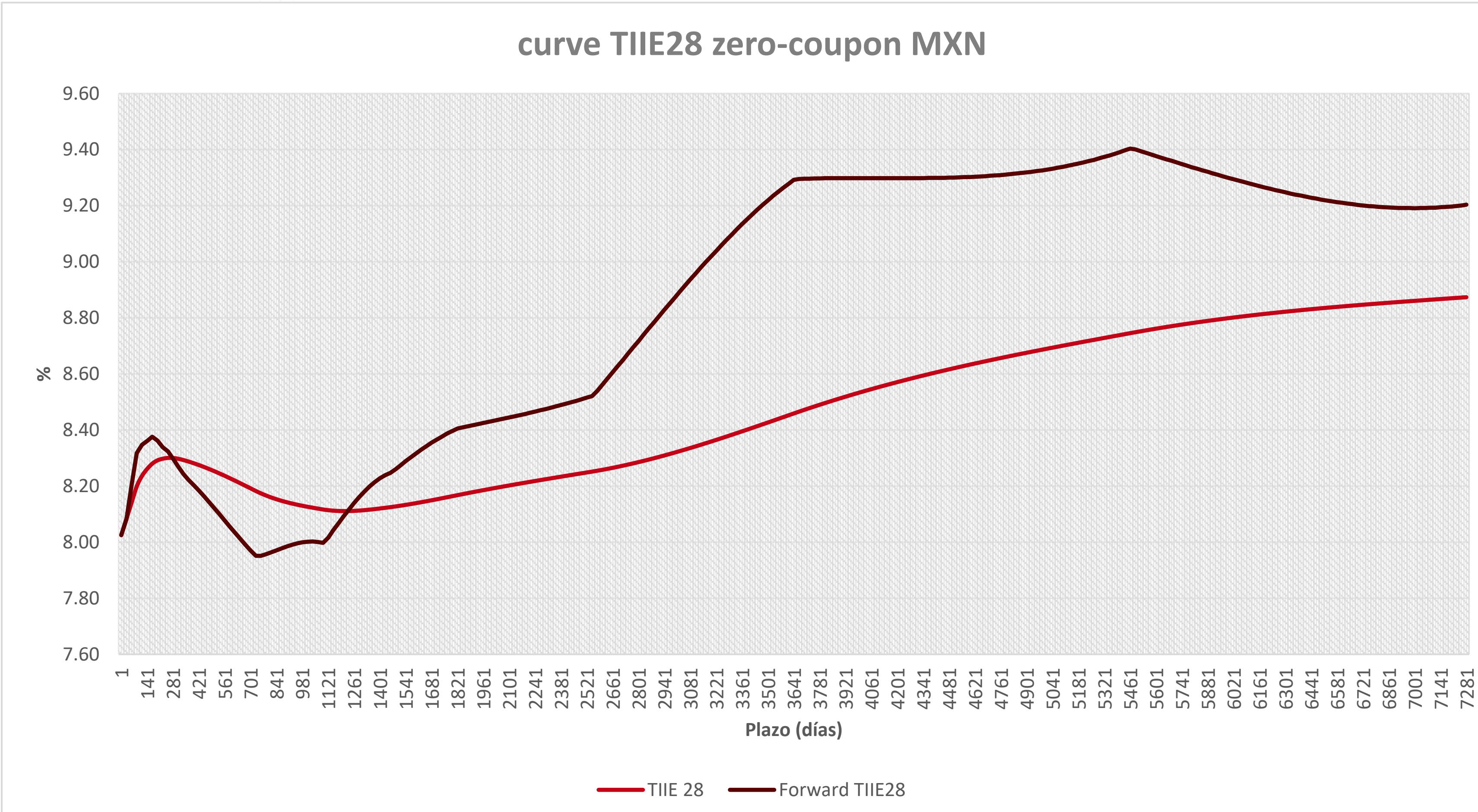
- Remembering:
  - The general idea of the single curve valuation approach is that the market value of all interest rate derivatives, denominated in the same currency, depends on a single curve, which is used, both as a discount curve and as a curve for the projection of future flows.
- The construction of this curve can be summarized in the following steps:
  - A finite set of series of a plain vanilla instrument that is traded on the market is selected, with maturities at different expirations.
  - The pair interest rates (yield) that are quoted in the money and derivatives market are used.
  - By means of the bootstrapping technique, discount coupon rates of lower maturity are calculated up to those of greater maturity. From the spot zero coupon curves, the forward interest rates that are used to project the cash flows are estimated.

# IV. Single curve approach

- **Exercise.** Build the zero-coupon curve for the MXN case.
  - The curve will be constructed from the IRS prices on TIIE 28. They are the derivative instruments on the most traded MXN currency interest rates and have maturities of up to 30 years.
  - For the purposes of the exercise, the zero-coupon curve will be obtained, with terms of up to 2 years, with information from September 10, 2018.
  - The market prices and the characteristics of the IRS on TIIE28 are:

| Expiration | Yield | Kind |
|---|---|---|
| 1D | 7.75 | Cash |
| 28D | 8.1149 | Cash |
| 84D | 8.2 | Swap |
| 168D | 8.28 | Swap |
| 252D | 8.3 | Swap |
| 364D | 8.2875 | Swap |
| 728D | 8.1875 | Swap |

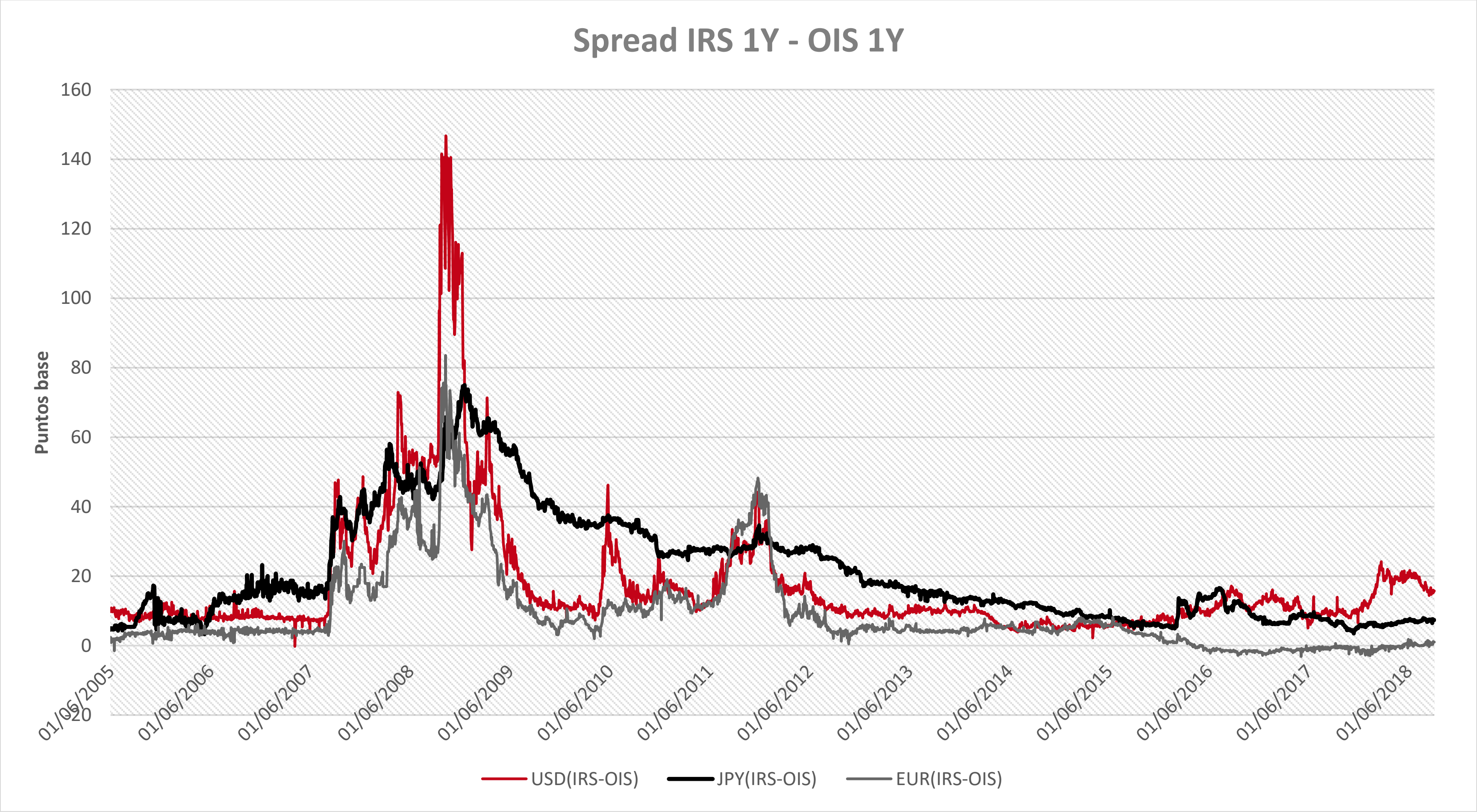
# IV. Single curve approach



# V. Multi-curve approach

- Remembering:
  - It became more important to "collateralize" OTC contracts in order to reduce the credit risk involved in bilateral transactions.
  - The multi-curve framework was introduced for the purpose of valuating derivatives with collateral.
  - With the crisis, Ibor rates stopped being a good proxy for risk-free rates. And opted for OIS rates.
- Before the financial crisis, the Libor-OIS spread in dollars was on average 8.86 basis points, that is, there was no significant difference in discounting derivatives with a Libor curve instead of an OIS curve.
- From August 2007 to December 2008, the average differential was approximately 54 basis points. The crisis produced an increase in this differential as a consequence of the deterioration of bank credit quality and fear of uncertainty.

# V. Multi-curve approach



# V. Multi-curve approach

- An "Overnight Indexed Swap" (OIS) is a fixed-floating interest rate swap, where the floating leg is linked to an overnight rate index, eg. Fed Funds or EONIA.
- There are several reasons for using overnight rates as risk-free rates. Highlights:
  - They are based on real market operations, calculated by the average rate at which these transactions occur.
  - The OIS market is active and liquid in different currencies. It has maturities of up to 30 years.
  - ISDA contracts generally use these rates as the cash collateral rate.
  - It reflects the cost of funding federal funds and treasury bonds.

# V. Multi-curve approach

- The OIS are totally collateralized, and the collateral coin coincides with the currency of the overnight rate.
- Under the assumption that the collateral rates are equal to the overnight rates, the discount curve can be constructed using the OIS.
- In the multi-curve framework, obtaining the discount and forward curves can be summarized in the following steps:
  - Decide what is the appropriate discount rate of the derivatives that are going to be valued.
  - Select the market instruments corresponding to that discount rate and create a zero-coupon curve with the single curve methodology.
  - Select a set of a plain vanilla interest rate instrument traded in the market with increasing maturities.
  - Construct the forward curves, using the selected set, using bootstrapping and the obtained discount curve.

# V. Multi-curve approach

- **Exercise.** Build zero-coupon discount and projection curves for the MXN case.
- Considerations:
  - In the Mexican market the overnight rate is the bank funding rate; however, there are no quotes of OIS instruments in MXN.
  - Derivatives on interest rates are collateralized mainly in other currencies.
  - You have to use a methodology for the construction of the curves with the aforementioned restrictions and replicate the market quotations considering the main currency of the CSA.
  - In order to show the multi-curve construction methodology, a two-step bootstrapping will be implemented to construct the forward MXN curve (TIIE 28d as index) and the MXN discount curve with collateral in EUR.

# V. Multi-curve approach

- Process:
  - Select a convenient plain vanilla derivative instrument set:

| IRS on TIIE28 | Input | Status | Source |
|---|---|---|---|
| Fixed leg | Yield rate | Known | Market price |
| | Discount curve MXN with collateral in EUR | Not known | Implicitly estimated |
| Floating leg | Implicit zero curve TIIE28 to estimate forward rates | Not known | Implicity estimated |
| | Discount curve MXN with collateral in EUR | Not known | Implicity estimated |

| CCS USD vs EUR | Input | Status | Source |
|---|---|---|---|
| MXN leg | Zero curve TIIE28 to estimate forward rates. | Not known | Implicity estimated |
| | Discount curve MXN with collateral in EUR | Not known | Implicity estimated |
| EUR leg | Basis Swaps | known | Market price |
| | Discount curve EUR with collateral EUR | known | Implicity estimated |
| | Curve zero Euribor 1m to estimate forward rates | known | Implicity estimated |

# V. Multi-curve approach

- In the case of the cross currency swaps, the discount curves EUR with collateral in EUR and Euribor 1m projection curve for the leg in EUR should be taken as inputs. The curves are constructed from the quoted prices of the following instruments:

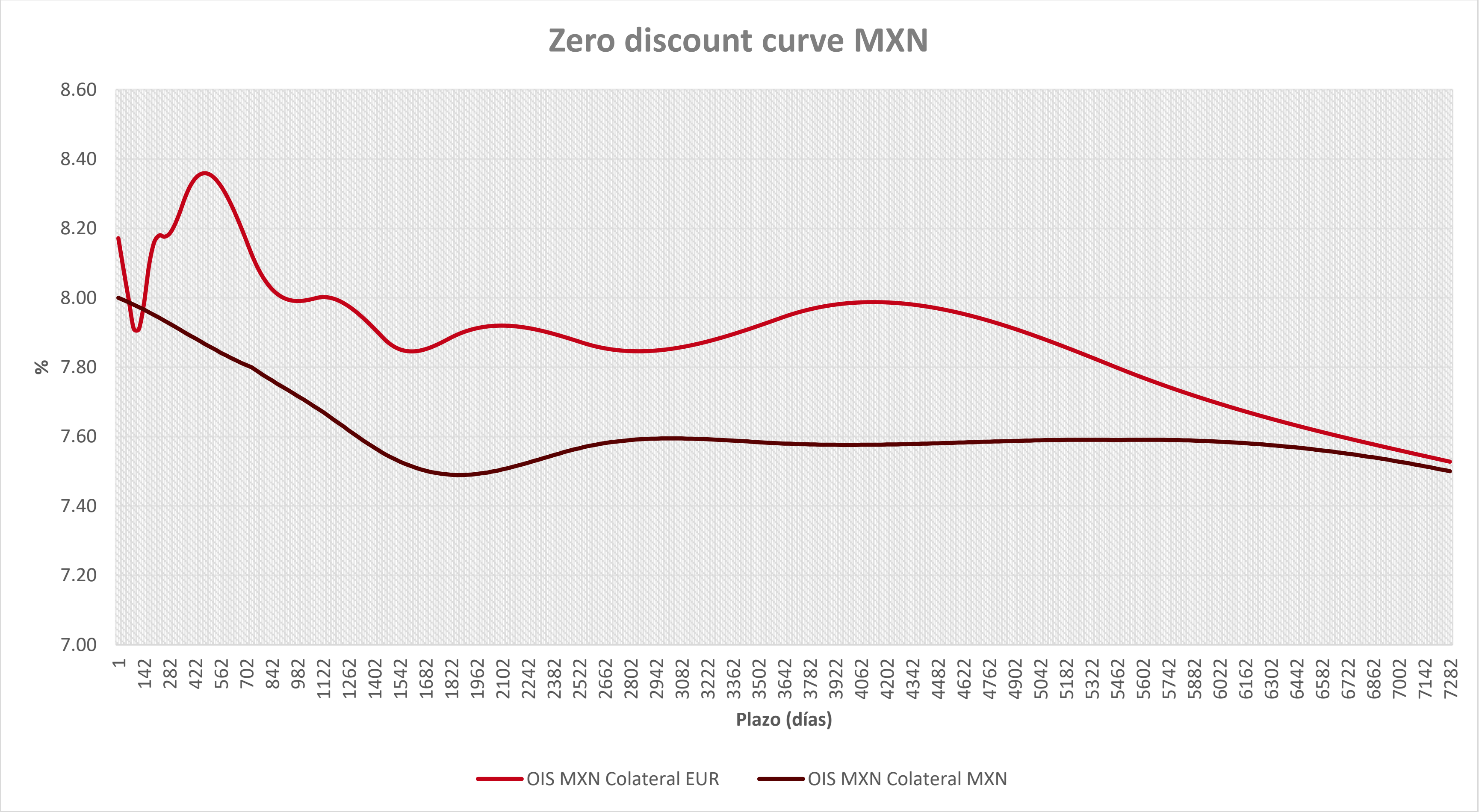
| OIS EUR | Input | Status | Source |
|---|---|---|---|
| Fixed leg | Yield rate | Known | Market price |
| | Discount curve EUR with collateral EUR | Not known | Implicity estimated |
| Floating leg | Discount curve EUR with collateral EUR | Not known | Implicity estimated |
| | Discount curve EUR with collateral EUR | Not known | Implicity estimated |

| IRS on Libor EUR 1m | Input | Status | Source |
|---|---|---|---|
| Fixed leg | Yield rate | Known | Marketing price |
| | Discount curve EUR with collateral EUR | Known | Implicity estimated |
| Floating leg | Curve zero Euribor 1m to estimate forward rates | Not known | Implicity estimated |
| | Discount curve EUR with collateral EUR | Known | Implicity estimated |

# V. Multi-curve approach

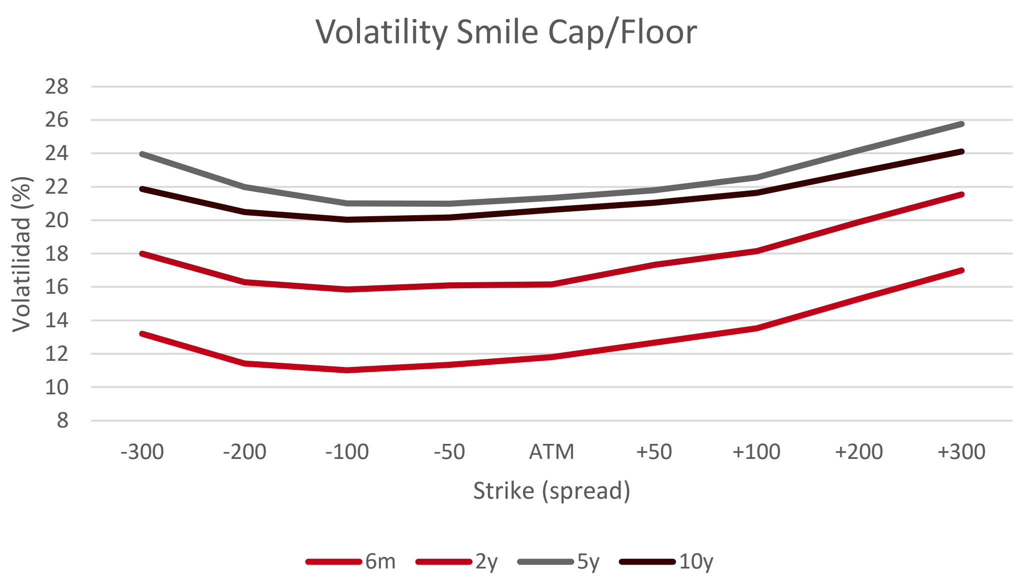
- Build each of the unknown curves in the following order (for illustrative purposes the curve will be built with a term of up to 2 years and with information as of September 10, 2018):
  - Get the zero-discount coupon curve EUR with collateral EUR.
  - Obtain the zero-coupon curve implied Euribor 1m.
  - Obtain the TIIE 28 zero coupon curves and the MXN discount curve with collateral in EUR, applying the two-step bootstrapping methodology, so that a parallel calibration is performed.

# V. Multi-curve approach



# VI. SABR Model

- In the original Black-Scholes model, it is assumed that volatility is constant; however, it is often observed that options with different strikes have different implied volatilities (which are more expensive when they are “in” and “out” of money) and this is known as "volatility skew" or "volatility smile".



# VI. SABR Model

- The SABR model captures the correct form of the implicit volatility curve observed in the market, as well as the correct dynamics of the smile.
- SABR is an acronym for Stochastic, Alpha, Beta, Rho. The formula for estimating ATM volatility is:

$$\sigma_{\text{ATM}} = \frac{\alpha}{f^{(1-\beta)}}\left\{1 + \left[\frac{(1-\beta)^2}{24}\frac{\alpha^2}{f^{2-2\beta}} + \frac{1}{4}\frac{\rho\beta\nu\alpha}{f^{(1-\beta)}} + \frac{2-3\rho^2}{24}\nu^2\right]t_{ex} + \ldots\right\}$$

- The model does not provide the prices of the options exactly, but gives an estimate of the implied volatility curve.
- Knowing the set of SABR parameters, the valuation of the options is reduced to the calculation of the effective volatility by means of the model, to then use it in Black's formula.

# VI. SABR Model

- Therefore, the "best" set of SABR parameters must be chosen. These parameters must be estimated, based on the calibration of the market data of the cap/floor and/or swaptions.
- The calibration is done through an iterative process; it starts from a set of parameters with initial values, which are substituted in the equation of the SABR model, and each parameter varies until finding the values of the parameters that, together, allow the estimated volatilities to be as close as possible to the market volatilities.
  - The prices quoted in the market of derivative instruments, the different strikes and terms are obtained.
  - Once the prices are known, the implied volatilities are estimated, clearing them of Black's formula. As a result, a surface of market volatilities is obtained.

# VI. SABR Model

- Subsequently, the parameters α, β, ρ and ν are found for each smile within the surface, that is, for the set of options according to their term. Likewise, the ATM forward rates of each set of options are estimated.

- In the case of surfaces in MXN, the following is considered:
  - It is assumed that the cap / floor and swaptions prices are in MXN under a CSA contract with collateral, so the zero-coupon curves calibrated according to the collateral currency must be used.
  - Forward rates are obtained from adjusted market zero-coupon curves depending on the collateral currency of the derivative instrument used for its estimation.

# VI. SABR Model

- Parameters:

| Plazo | alpha | beta | rho | nu |
| --- | --- | --- | --- | --- |
| 6m | 0.11762 | 0.99958 | 0.46229 | 0.53577 |
| 1y | 0.11630 | 0.99983 | 0.46228 | 0.52914 |
| 2y | 0.15409 | 0.99803 | 0.36495 | 0.57549 |
| 3y | 0.15281 | 0.99473 | 0.25223 | 0.65336 |
| 4y | 0.16306 | 0.99541 | 0.24564 | 0.60165 |
| 5y | 0.18365 | 0.99732 | 0.22515 | 0.56945 |
| 7y | 0.17499 | 0.97993 | 0.24356 | 0.52357 |
| 10y | 0.16853 | 1.00000 | 0.25263 | 0.46563 |
| 15y | 0.15218 | 0.97867 | 0.26082 | 0.43776 |

# VI. SABR Model

**BLOOMBERG**

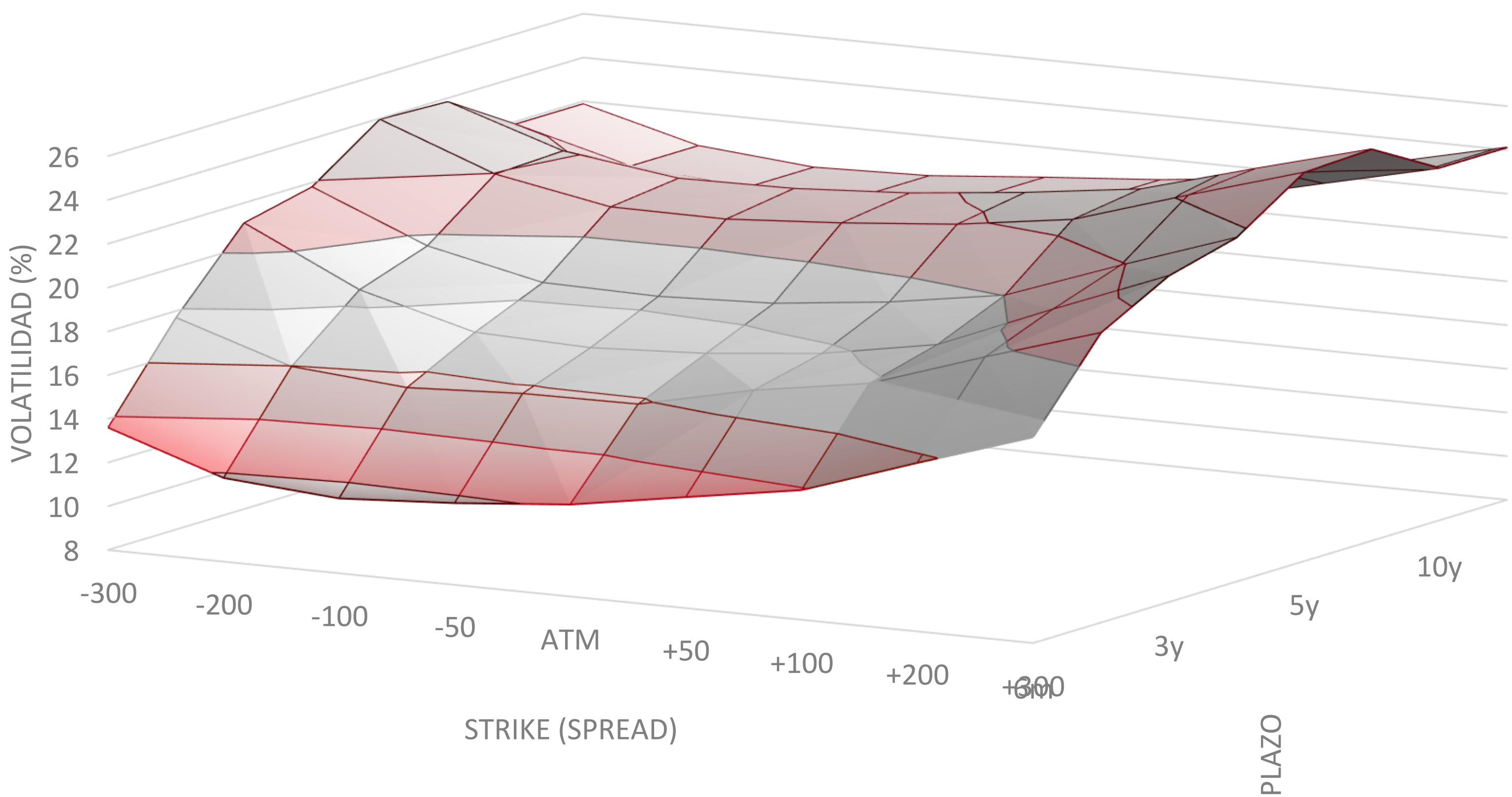
| Plazo | Strikes 5.0% | 6.0% | 7.0% | 8.0% | ATM | 9.0% | 10.0% | 11.0% | 12.0% |
|---|---|---|---|---|---|---|---|---|---|
| 6Mo | 18.55 | 17.55 | 16.72 | 16.16 | 16.34 | 15.66 | 15.25 | 14.99 | 14.78 |
| 1Yr | 18.69 | 17.6 | 16.73 | 16.16 | 15.97 | 15.67 | 15.26 | 15.01 | 14.81 |
| 2Yr | 27.09 | 21.59 | 18.98 | 17.64 | 17.44 | 17.01 | 16.74 | 16.59 | 16.43 |
| 3Yr | 25.74 | 21.69 | 19.56 | 18.31 | 18.28 | 17.65 | 17.31 | 17.04 | 16.76 |
| 4Yr | 23.17 | 20.32 | 18.82 | 17.85 | 17.95 | 17.24 | 16.81 | 16.42 | 16.02 |
| 5Yr | 22.27 | 19.64 | 18.27 | 17.38 | 17.58 | 16.75 | 16.24 | 15.76 | 15.3 |
| 7Yr | 21.79 | 19.2 | 17.83 | 16.94 | 17.05 | 16.28 | 15.71 | 15.19 | 14.72 |
| 10Yr | 22.06 | 19.5 | 18.05 | 17.12 | 17.51 | 16.43 | 15.88 | 15.41 | 15 |
| 15Yr | 23.83 | 20.91 | 19.13 | 17.98 | 17.84 | 17.19 | 16.61 | 16.13 | 15.74 |

**VAR**

| Plazo | Strikes (Spreads en pb) -300 | -200 | -100 | -50 | ATM | +50 | +100 | +200 | +300 |
|---|---|---|---|---|---|---|---|---|---|
| 6M | 13.592 | 11.821 | 11.416 | 11.734 | 12.2 | 13.071 | 13.921 | 15.695 | 17.4 |
| 1Y | 13.592 | 11.821 | 11.416 | 11.734 | 12.2 | 13.071 | 13.921 | 15.695 | 17.4 |
| 2Y | 17.69 | 15.98 | 15.545 | 15.804 | 15.85 | 17.019 | 17.855 | 19.6 | 21.24 |
| 3Y | 21.08 | 18.555 | 17.141 | 16.963 | 17.232 | 17.758 | 18.693 | 20.842 | 22.9 |
| 4Y | 21.794 | 19.627 | 18.479 | 18.378 | 18.589 | 19.177 | 20.02 | 21.915 | 23.71 |
| 5Y | 23.95 | 21.976 | 21.005 | 20.975 | 21.331 | 21.798 | 22.547 | 24.189 | 25.75 |
| 7Y | 23.826 | 22.104 | 21.38 | 21.442 | 21.763 | 22.316 | 23.005 | 24.474 | 25.89 |
| 10Y | 21.872 | 20.485 | 20.025 | 20.164 | 20.623 | 21.036 | 21.636 | 22.891 | 24.11 |
| 15Y | 21.872 | 20.485 | 20.025 | 20.164 | 20.623 | 21.036 | 21.636 | 22.891 | 24.11 |

# VI. SABR Model

## Volatility surface of Cap/Floor, USD CSA – VaR



# VI. SABR Model

## Surface volatility Cap/Floor – Bloomberg